expected from a production cable. For the aforementioned cases, the worst unbalance observed was 0.95 pF/m when one diameter was increased by 10 percent over its nominal size and moved 10 percent farther away from the axis of the shield than its nominal position. The Fourier coefficients of the surface charge densities (not presented, cf. [5]) are required in determining the propagation constants and associated propagation modes that can be expected on a production cable.

VI. Conclusion

In this short paper the capacitance matrix of a straight pair of wires in a shield was determined theoretically. The Fourier coefficients of the surface change densities on the inner conductors and the shield and the various capacitances associated with the cable structure were then determined. The voltage excitations were completely arbitrary and the cable structure was unbalanced in the sense that the wires can be asymmetrically located about the axis of the shield and can have different radii. The theoretical results were evaluated numerically for the case of a shielded pair cable modified to reflect possible inaccuracies in the nominal size and spacing of the wires due to inaccuracies in the manufacturing process.

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Equivalent Circuits and Characteristics of Inhomogeneous Nonsymmetrical Coupled-Line Two-Port Circuits

VIJAI K. TRIPATHI, MEMBER, IEEE

***Abstract*—The equivalent circuits and the properties of nonsymmetrical coupled-line two-port prototype circuits in an inhomogeneous medium are presented. Potential applications include both symmetrical and nonsymmetrical circuits for narrow- and wide-band applications as filters and impedance transformers.**

Introduction

Symmetrical and nonsymmetrical two-port circuits consisting of nonsymmetrical uniformly coupled lines in an inhomogeneous medium [1] may be designed for various applications as filters and impedance matching networks by utilizing the coupled-line four-port parameters and resulting equivalent circuits for various structures. Equivalent circuits and some characteristics of identical coupled lines in an inhomogeneous medium have been obtained by Zysman and Johnson [2]. Allen [3] has utilized these to formulate the design procedures for various two-port circuits for application as filters for the case of large mode velocity ratios. For the general case of lossless quasi-TEM coupled lines in an inhomogeneous medium, the four-port equivalent circuit in a graph equivalent form [4] has been obtained by Costamagna and Maltese [5], and the equivalent circuit consisting of uncoupled normal mode lines with mode decoupling and coupling networks consisting of ideal transformer banks has been given by Chang [6]. In addition, the equivalent circuits for various prototypes for the special case of congruent nonsymmetrical coupled lines, where the line parameters are such that the normal modes degenerate into an even-voltage and an odd-current mode [1], have been derived by Allen [7]. In this short paper, the properties of nonsymmetrical coupled lines in an inhomogeneous medium and their four-port parameters as obtained by Tripathi [1] are used to obtain the equivalent circuits and characteristics of various prototypes for the case of quasi-TEM lossless lines. Such structures offer inherent impedance transforming capability and added flexibility in design through an additional variable as compared to identical coupled lines in an inhomogeneous medium.

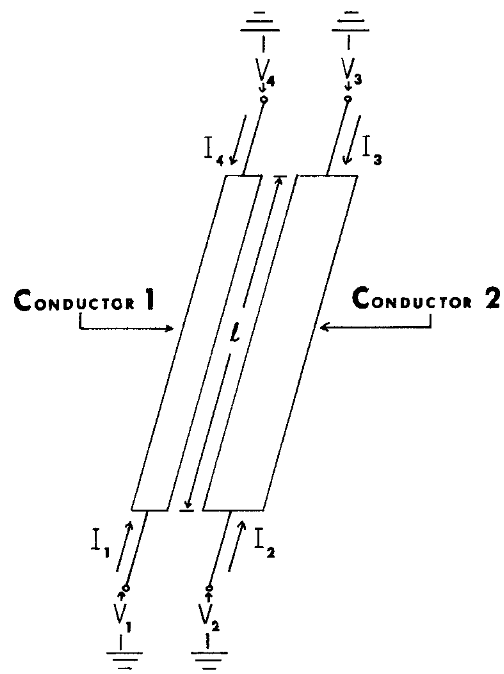

Fig. 1. Schematic of a nonsymmetrical coupled-line four-port in an inhomogeneous medium.

The immittance matrix elements for the coupled-line four-port (Fig. 1) have been derived in terms of the normal mode parameters of the coupled system [1]. These parameters are the propagation constants, the mode voltage ratios on the two lines, and the partial mode impedances and admittances of the two lines for the normal modes of the coupled system. For the case of lossless coupled lines, characterized by their self- and mutual inductances per unit length and capacitances per unit length as given by L_1, L_2, L_m and C_1, C_2, C_m where L_j and C_j ($j = 1,2$) are self-inductance and capacitance per unit length of line j in presence of line k ($k = 1,2$: $k \neq j$), and L_m and C_m are mutual

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The author is with the Department of Electrical and Computer Engineering, Oregon State University, Corvallis, OR 97331.

inductance and capacitance per unit length, respectively, for the quasi-TEM case, these normal mode parameters are given by

$$\beta_{c,\pi} = \frac{\omega}{\sqrt{2}} [L_1C_1 + L_2C_2 - 2L_mC_m \pm \sqrt{(L_2C_2 - L_1C_1)^2 + 4(L_mC_1 - L_2C_m)(L_mC_2 - L_1C_m)}]^{1/2} \quad (1)$$

$$R_{c,\pi} \triangleq \frac{V_2}{V_1}, \quad \text{for } \beta = \pm\beta_c \text{ and } \pm\beta_\pi, \text{ respectively}$$

$$= \frac{L_2C_2 - L_1C_1 \pm \sqrt{(L_2C_2 - L_1C_1)^2 + 4(L_mC_1 - L_2C_m)(L_mC_2 - L_1C_m)}}{2(L_mC_2 - L_1C_m)} \quad (2)$$

$$Z_{c1} = \frac{\omega}{\beta_c}\left(L_1 - \frac{L_m}{R_\pi}\right) = \frac{\beta_c}{\omega}\left(\frac{1}{C_1 - R_cC_m}\right) = \frac{1}{Y_{c1}} \quad (3a)$$

$$Z_{\pi 1} = \frac{\omega}{\beta_\pi}\left(L_1 - \frac{L_m}{R_c}\right) = \frac{\beta_\pi}{\omega}\left(\frac{1}{C_1 - R_\pi C_m}\right) = \frac{1}{Y_{\pi 1}} \quad (3b)$$

$$Z_{c2} = -R_cR_\pi Z_{c1} = \frac{1}{Y_{c2}} \quad (3c)$$

$$Z_{\pi 2} = -R_cR_\pi Z_{\pi 1} = \frac{1}{Y_{\pi 2}} \quad (3d)$$

where $\beta_{c,\pi}$ are the phase constants for the two modes, $R_{c,\pi}$ are the ratio of the voltages on the lines for the two normal modes, $Z_{c1}, Z_{\pi 1}, Z_{c2}, Z_{\pi 2}$ are the mode impedances of the two lines, and $Y_{c1}, Y_{\pi 1}, Y_{c2}, Y_{\pi 2}$ are the corresponding admittances.

Two-Port Circuits

In terms of these normal mode parameters, various two-port prototypes and their equivalent circuits are given in Table I. For a given set of boundary conditions at any two ports of the coupled-line four-port, these equivalent circuits are derived in a straightforward manner by utilizing known procedures. These include the direct use of the immittance matrices [1], [2], [7] or the use of either the graph equivalent circuit representing the admittance matrix [4], [5] or the normal mode four-port equivalent circuit [6].

The equivalent circuits for cases a–f, where the boundary conditions are the same at any given two-ports of the structure, the impedance, or the admittance matrix of the coupled line four-port [1], lead directly to the equivalent circuits for structures with open- or short-circuit boundary conditions, respectively.

The graph equivalent circuit representing the admittance matrix of the coupled-line four-port may also be used to find these and other equivalent circuits, and is particularly suited for circuits where the ports are interconnected or terminated in known impedances. Examination of the admittance matrix of the coupled-line four-port reveals that it can be expressed as $[Y] = [Y_c] + [Y_\pi]$ with $[Y_c]$ and $[Y_\pi]$ having the same form as the admittance matrix of a coupled-line four-port in a homogeneous medium [4]. This leads to the equivalent circuit for the coupled-line four-port in an inhomogeneous medium as a parallel combination of the equivalent graph representation [4] for the two modes as given by Costamagna and Maltese [5]. The equivalent circuits for cases g and h are derived by utilizing the graph equivalent circuit. It should be noted here that Costamagna and Maltese have derived the graph equivalent circuits for short-circuited interdigital and comb sections which can be easily reduced to the equivalent circuits of cases d and f, respectively, in Table I by utilizing the relationships between the mode admittance parameters as given by (3).

Another useful method for deriving the equivalent circuits, particularly for those having mixed boundary conditions (one port open, the other shorted), involves the use of the equivalent normal mode four-port circuit consisting of uncoupled normal mode lines with decoupling and coupling networks [6]. These decoupling and coupling networks consist of ideal transformer banks and represent the linear transformations interrelating the line voltages and currents to the mode voltages and currents, e.g., $V_{c,\pi} = V_2 - R_{\pi,c}V_1$ and $I_{c,\pi} = I_2 + I_1/R_{c,\pi}$ [1]. The equivalent circuits for cases i and j in Table I are derived by utilizing the normal mode four-port equivalent circuit.

The equivalent circuits given in Table I reduce to the known circuits for symmetric inhomogeneous cases [2], [6] where $R_c = -R_\pi = 1$, and congruent nonsymmetrical inhomogeneous cases [7] where $R_c = 1$ and $R_\pi = -(C_1 - C_m)/(C_2 - C_m)$.

The properties of various structures can be evaluated by utilizing their equivalent circuits or terminal circuit parameters. For example, the insertion loss (IL) for the open-circuited symmetric section (case a) terminated by Z_0 at both the input and output end is found to be

$$\text{IL} = 10 \log_{10}\left[1 + \frac{1}{4}\left\{\frac{(Z_1^2 + Z_2^2 - Z_0^2)\sin\theta_c \sin\theta_\pi + 2Z_1Z_2(1 - \cos\theta_c\cos\theta_\pi)}{Z_0(Z_1 \sin\theta_\pi + Z_2\sin\theta_c)}\right\}^2\right] \quad (4)$$

where

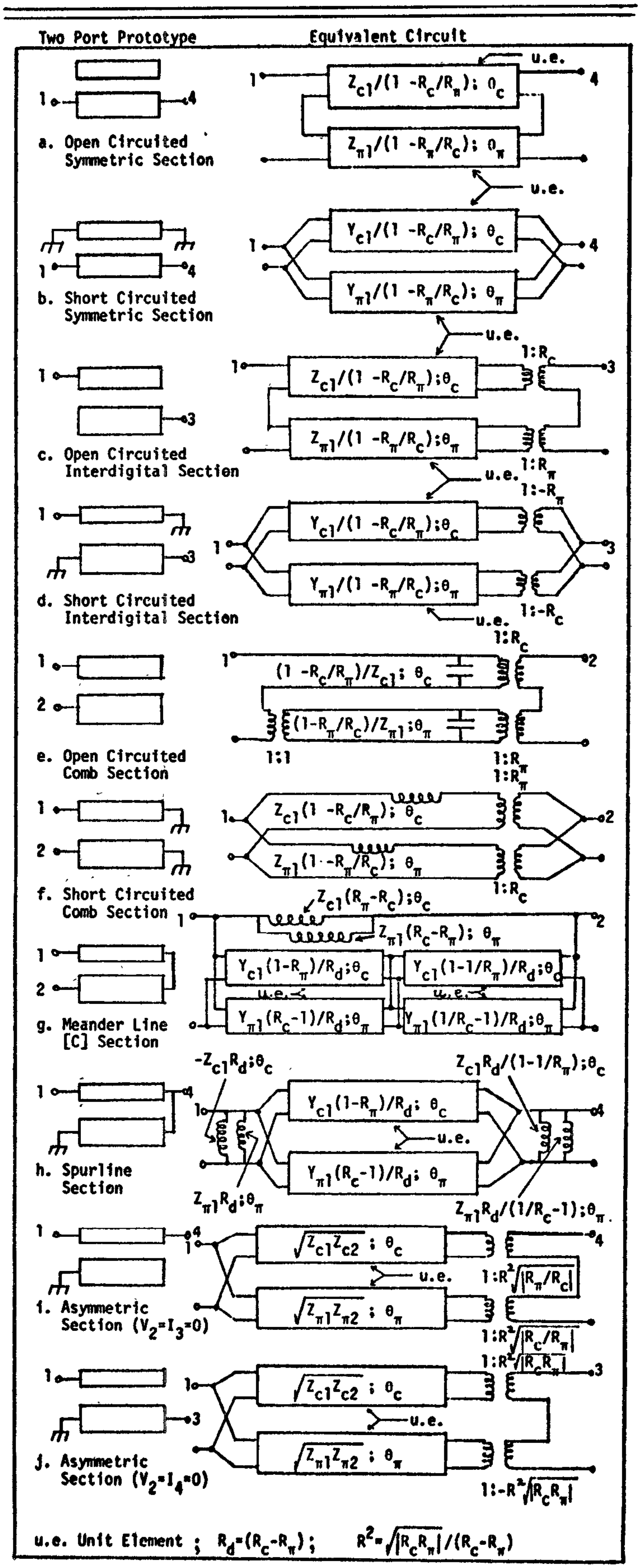
$$Z_1 = Z_{c1}/(1 - R_c/R_\pi)$$

$$Z_2 = Z_{\pi 1}/(1 - R_\pi/R_c).$$

This function is of the same form as that obtained by Allen [3] for a structure consisting of identical coupled lines resulting in similar filter properties. For the case of large mode velocity ratio, multiple poles in the stopband and zeros in the passband may be realized using a single section. For the nonsymmetrical coupled-line system, an additional variable is now available yielding more flexibility in the design of the structure. The IL function for the open-circuited symmetrical section discussed previously, and a nonsymmetrical interdigital section (case c, Table I) is shown in Figs. 2 and 3 for some typical cases to demonstrate their characteristics for possible applications as filters and transformers.

The physical geometry of the structure for the case of nonsymmetrical broadside coupled lines can be obtained by utilizing the results of Chao [8], who has formulated the procedure for calculating self- and mutual capacitances.

TABLE I



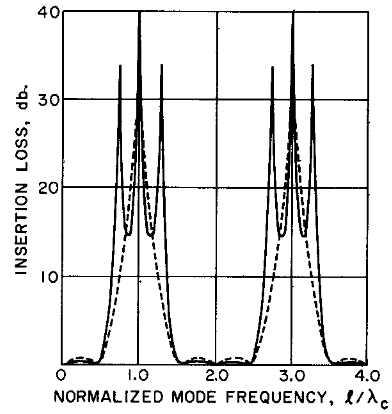


Fig. 2. Frequency response of open-circuited symmetric section. ——— $Z_{c1} = 60\ \Omega$, $Z_{\pi 1} = 22.5\ \Omega$, $\beta_c = 2\beta_\pi$, $R_c = 1.5$, $R_\pi = -0.75$; 50-Ω terminations. - - - - $Z_{c1} = 33\ \Omega$, $Z_{\pi 1} = 30\ \Omega$, $\beta_c = 2\beta_\pi$, $R_c = 0.75$, $R_\pi = -1.5$; 50-Ω terminations.

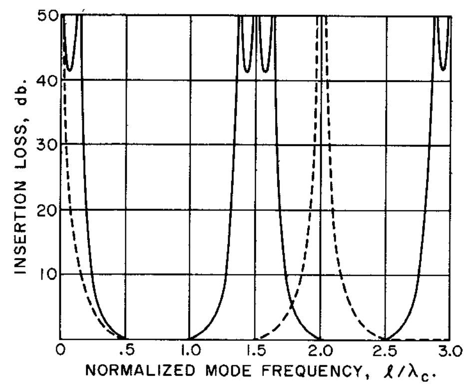


Fig. 3. Frequency response of open-circuited interdigital section. ——— $Z_{c1} = 60\ \Omega$, $Z_{\pi 1} = 30\ \Omega$, $R_c = 1$, $R_\pi = -2$, $\beta_c = 3\beta_\pi$, $Z_{in} = 50\ \Omega$, $Z_{out} = 78\ \Omega$. - - - - $Z_{c1} = 60\ \Omega$, $Z_{\pi 1} = 30\ \Omega$, $R_c = 1$, $R_\pi = -2$, $\beta_c = 2\beta_\pi$, $Z_{in} = 50\ \Omega$, $Z_{out} = 78\ \Omega$.

Conclusion

The equivalent circuits for various nonsymmetrical coupled-line circuits in an inhomogeneous medium derived in the short paper may be used to formulate design techniques for a host of applications, including those which cannot be realized using identical coupled lines, such as matching networks. The typical response functions presented for an open-circuited symmetrical section and an open-circuited nonsymmetrical interdigital section demonstrate their applications as narrow- and wide-band low-pass, bandpass, and band rejection filters and transformers.

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